MATHEMATICS

REVISION BOOKLET
2025 TERM 3

Grade 12

This revision program is designed to assist you in revising the critical content and skills covered during the 3nd term. The purpose is to prepare you to understand the key concepts and to provide you with an opportunity to establish the required standard and the application of the knowledge necessary to succeed in the NCS examination.

The revision program covers the following topics:

- Financial Mathematics (± 15 of 150 marks in Paper 1)
- Probability (± 15 of 150 marks in Paper 1)
- Statistics (± 20 of 150 marks in Paper 2)
- Trigonometry(± 50 of 150 marks in Paper 2)

Dear Learner

In the Term 3 revision material you will be revising:

- Financial Mathematics: ± 15 of 150 marks of Paper 1
- Probability: ± 15 of 150 marks of Paper 1
- Statistics: ± 20 of 150 marks of Paper 2
- Trigonometry: ±50 of 150 marks of Paper 2

Each of the above topics starts with a summary of essential is compiled as follows:

- Summary of the key concepts in the topic.
- Section A, essentially questions which you must ensure you are able to do, to at least obtain 30% in Mathematics.
- Section B, these are questions at a higher level

Please ensure that you work thoroughly through the summary of key concepts at the beginning of each topic. This is essential before embarking on section A and B. If anything in the notes are not understood it is important that you get support with it. Any challenges experienced indicates some shortcoming in the understanding of certain concepts which will hamper the completion of questions on the topic. Ensure that you get support from a tutor, teacher, consult your textbook or search online for support. You could also use the Tutonic concept videos to support with your understanding of certain concepts.

By clicking on the link, https://westerncape-my.sharepoint.com/:b:/g/personal/razzia_ebrahim_westerncape_gov_za/Ecexc5QG6-1IocxgbmJkJDQBh5dnnR00eqzJm0hfNuAJiw?e=AWtIIU you will access a pdf with mindmaps on all the different topics in grade 12.

To get support on a given topic,

1. Goto the mindmap on that topic.
2. The mindmap reflects sub-topics from grade 10 – 12. Click on the ball next to the subtopic to access the video explaining that concept.

After having worked through the topic summary start doing the questions on the topic. Please check your answers as you progress through the material. All answers are available in the Tutor Guide. It is important to know whether your answers are correct before progressing to the next question. You will learn a lot by looking at the solutions also look at how solutions are set out. If you are not able to even start a particular question, look at the memorandum, it is all part of the learning process. Mark the question and make sure you attempt it again a day later. Please spend at least an 2 hours every day on revising Mathematics.

Thank you for working and revising your Mathematics daily in preparing for the examination. Working regularly at Mathematics is extremely important to realising your success in this subject.

All of the best!!!!

INDEX

Financial Mathematics

FINANCE – SUMMARY

FORMULES	
Simple Interest	$A = P(1 + in)$
Compound Interest	$A = P(1 + i)^n$
Straight line depreciation	$A = P(1 - in)$
Reducing Balance depreciation	$A = P(1 - i)^n$
Effective and Nominal interest	$1 + i_{eff} = \left(1 + \frac{i_{nom}}{m}\right)^m$
Sum of Geometric series	$S_n = \frac{a(r^n-1)}{r-1}$ or $S_n = \frac{a(1-r^n)}{1-r}$ $, r \neq 1$
A	Final amount
P	Original amount
i	Is the interest rate per period
n	Is the number of periods
i_{eff}	Is the effective annual interest rate
i_{nom}	Nominal interest is the quoted annual interest rate
m	Is the number of times interest is compounded in one year.
Future value annuity	$F = \frac{x[(1+i)^n-1]}{i}$
Present value annuity	$P = \frac{x[1-(1+i)^{-n}]}{i}$
n	Is the number of payments

NOTE:

- Questions in this revision were compiled from past exam National papers for GR 12.

Section A:

QUESTION 1

1.1 Anisha and Lindiwe each received R12 000 to invest for a period of 5 years. They invested the money at the same time according to the following options:

- Anisha: 8,5% p.a. simple interest. At the end of the 5 years she will receive an additional bonus pay-out of exactly 7,5% of the original amount invested.
- Lindiwe: 8,5% p.a. compounded quarterly.

Who will have the larger final amount after 5 years? Justify your answer with appropriate calculations. (6)

1.2 A company bought office furniture that cost R120 000. After how many years will the furniture depreciate to a value of R41 611,57 according to the reducing-balance method, if the rate of depreciation is 12,4% p.a.? (4)

1.3 Tebogo opened a savings account with a single deposit of R5 000 at the beginning of June 2015. He then made 24 monthly deposits of R800 at the end of every month, starting at the end of June 2015. The account earned interest at 15% p.a. compounded monthly.
Calculate the amount that should be in his savings account immediately after he makes the last deposit. (5)
[15]

QUESTION 2

2.1 Exactly five years ago Mpume bought a new car for R145 000. The current book value of this car is R72 500. If the car depreciates by a fixed annual rate according to the reducing-balance method, calculate the rate of depreciation. (3)

2.2 Samuel took out a home loan for R500 000 at an interest rate of 12% per annum, compounded monthly. He plans to repay this loan over 20 years and his first payment is made one month after the loan is granted.

2.2.1 Calculate the value of Samuel's monthly instalment. (4)

2.2.2 Melissa took out a loan for the same amount and at the same interest rate as Samuel. Melissa decided to pay R6 000 at the end of every month. Calculate how many months it took for Melissa to settle the loan. (4)

2.2.3 Who pays more interest, Samuel or Melissa? Justify your answer. (2)
[13]

QUESTION 3

On 1 June 2016 a bank granted Thabiso a loan of R250 000 at an interest rate of 15% p.a. compounded monthly, to buy a car. Thabiso agreed to repay the loan in monthly instalments commencing on 1 July 2016 and ending 4 years later on 1 June 2020. However, Thabiso was unable to make the first two instalments and only commenced with the monthly instalments on 1 September 2016.

3.1 Calculate the amount Thabiso owed the bank on 1 August 2016, a month before he paid his first monthly instalment. (2)

3.2 Having paid the first monthly instalment on 1 September 2016, Thabiso will still pay his last monthly instalment on 1 June 2020. Calculate his monthly instalment. (4)

3.3 If Thabiso paid R9 000 as his monthly instalment starting on 1 September 2016, how many months sooner will he repay the loan? (5)

3.4 If Thabiso paid R9 000 as a monthly instalment starting on 1 September 2016, calculate the final instalment to repay the loan. (4)

[15]

QUESTION 4

4.1 How long would the price of an asset take to reduce by a third of its original value if it depreciates on a reducing balance at a rate of 4,7% p.a.? (4)

4.2 Lebogo bought a tractor for Rx on 1 April 2016.

- She will trade in this tractor when she replaces it with a similar one in 5 years' time on 1 April 2021.
- The tractor depreciates by 20% p.a. according to the reducing-balance method.
- The price of a similar tractor increases by 18% annually.
- Lebogo calculated that if she deposited R8 000 per month into a sinking fund, which paid interest at 10% p.a. compounded monthly, she would have enough money to cover the replacement cost of the tractor. She made the first deposit in this fund on 30 April 2016 and will continue to do so at the end of every month until 31 March 2021.

4.2.1 Determine, in terms of x, what the book value of the current tractor will be on 1 April 2021 (that is, 5 years after it was bought). Give your answer correct to FIVE decimal places. (2)

4.2.2 Determine, in terms of x, what the price of a similar new tractor will be on 1 April 2021. Give your answer correct to FIVE decimal places. (2)

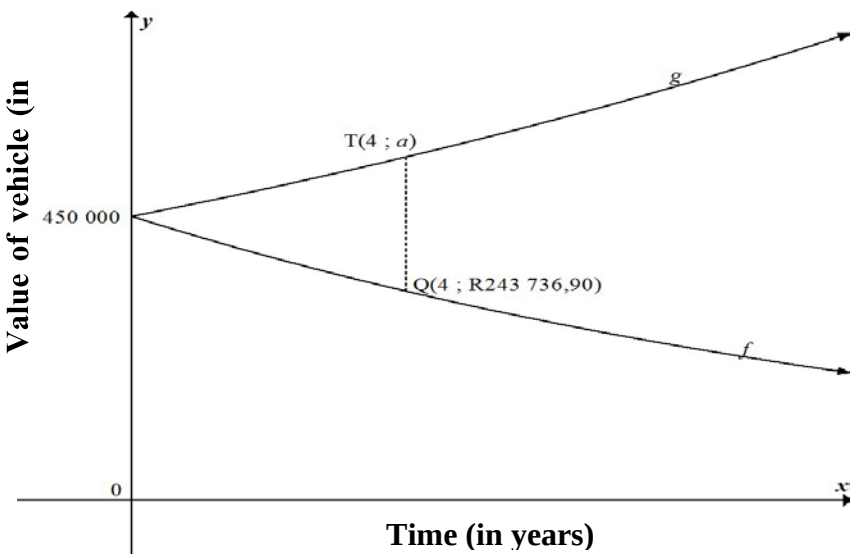
4.2.3 Calculate the amount accumulated in the sinking fund on 1 April 2021. (4)

4.2.4 Calculate the value of x, the price of the current tractor. Round off your answer to the nearest thousand (4)

[16]

QUESTION 5

The graph of f shows the book value of a vehicle x years after the time Joe bought it. The graph of g shows the cost price of a similar new vehicle x years later.



5.1 How much did Joe pay for the vehicle? (1)

5.2 Use the reducing-balance method to calculate the percentage annual rate of depreciation of the vehicle that Joe bought. (4)

5.3 If the average rate of the price increase of the vehicle is 8,1% p.a., calculate the value of a. (3)

5.4 A vehicle that costs R450 000 now, is to be replaced at the end of 4 years. The old vehicle will be used as a trade-in. A sinking fund is created to cover the replacement cost of this vehicle. Payments will be made at the end of each month. The first payment will be made at the end of the 13^{th} month and the last payment will be made at the end of the 48^{th} month. The sinking fund earns interest at a rate of 6,2% p.a., compounded monthly. Calculate the monthly payment to the fund. (5)

[13]

QUESTION 6.

Siphokazi bought a house. She paid a deposit of R102 000, which is equivalent to 12% of the selling price of the house. She obtained a loan from the bank to pay the balance of the selling price. The bank charges her interest of 9% per annum, compounded monthly.

6.1 Determine the selling price of the house. (1)

6.2 The period of the loan is 20 years and she starts repaying the loan one month after it was granted. Calculate her monthly instalment. (4)

6.3 How much interest will she pay over the period of 20 years? Round your answer correct to the nearest rand. (2)

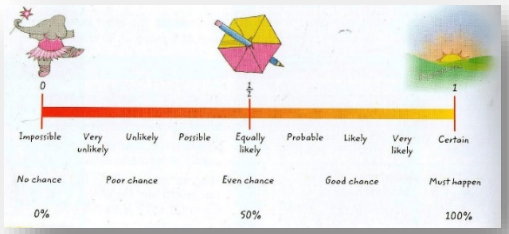
6.4 Calculate the balance of her loan immediately after her 85^{th} instalment. (3)

6.5 She experienced financial difficulties after the 85^{th} instalment and did not pay any instalments for 4 months (that is months 86 to 89). Calculate how much Siphokazi owes on her bond at the end of the 89^{th} month. (2)

6.6 She decides to increase her payments to R8 500 per month from the end of the 90^{th} month. How many months will it take to repay her bond after the new payment of R8 500 per month? (4)

[16]

Probability Revision

<table>
<tr>
<td>Probability of an event happening:
$P(A)$
$= \frac{number\ of\ possible\ times\ an\ event\ can\ happen}{number\ of\ possible\ outcomes}$
$= \frac{n(A)}{n(S)}$
[On Formula sheet!]</td>
<td> 
</td>
<td>Example:
Tossing a coin:

Outcome could be Heads or Tails

Therefore:
$P(\ Head) = \frac{1}{2}$</td>
</tr>
<tr>
<td>Independent events
The two events that do not affect each other’s outcomes.

Multiplication rule :
$P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B)$</td>
<td>There is a bag with 2 red and 3 blue balls
Event 1:
P(red) $= \frac{2}{5}$
Balls are replaced!
Event 2:
P(blue) = 3/5
P(Red and blue) $\frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$</td>
<td rowspan="2">The events A and B are independent. P(A) = 0, 4 and P (B) = 0, 5
Determine:
• P (A and B)
•
$P(A\ and\ B) = P(A) \times P(B)$
$= 0{,}4 \times 0{,}5$
$= 0{,}2$

• P (A or B)
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
$= 0.4 + 0{,}5 - 0{,}2$
$= 0{,}7$</td>
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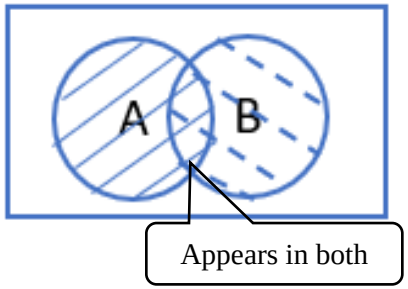
<tr>
<td>Mutually inclusive events:

A and B inclusive events as they have elements common.

Addition Rule:
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$</td>
<td> 
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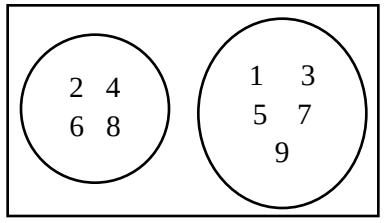
<tr>
<td>Mutually exclusive events
A and B mutually exclusive if they have no common elements.
Therfore: $P(A\ and\ B) = 0$
Addition rule becomes:
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
$P(A \text{ or } B) = P(A) + P(B) - 0$
$P(A \text{ or } B) = P(A) + P(B)$</td>
<td> 
</td>
<td>If events A and B are mutually exclusive and
• $P(A) = 0{,}1$
• $P(\ B) = 0{,}3$
Calculate $P(A\ or\ B)$
Solution: $P(A \text{ or } B) = P(A) + P(B)$
$= 0{,}1 + 0{,}3$
$= 0{,}4$</td>
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Complementary events

- T**he complementary rule:**

$P(not\ A) = P(A') = 1 - P(A)$

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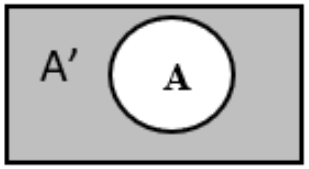

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Example:

The events A and B are independent. P(A) = 0, 4 and P (B) = 0, 5
Determine:

$$P(not\ A)$$
$$P(not\ A) = 1 - P(A)$$
$$= 1 - 0{,}4 = 0{,}6$$

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Counting Principle

The fundamental counting principle states:
Suppose there are *n* ways to make a choice, and for each of these there are *m* ways to make a second choice then the number of possible outcomes will be equal to $m \times n$ ways.

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Please note:

- If n items can be arranged in a row, then this can be done in $n!$ ways. (no repititions)

Note:
5! **5 factorial**
$5 \times 4 \times 3 \times 2 \times 1 = 5! = 120$

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Example:

How many different outfits could you put together with 4 shirts, 6 skits and 2 pairs of shoes?
Solution:
Number of outfits $= 4 \times 6 \times 2$
$= 48$ outfits

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Arrangements with repetition:

Arrangements with repetition:
$number\ of\ arrangements = k^x$
Where:
k is the number of choices
x is the number of times you can choose

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Example

How many three letter codes can be made from the letters in the word SPRING, if the letters can be repeated?

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Solution:

Number of ways
$5 \times 5 \times 5 = 5^3 = 125$

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Arrangements without repetition:

Arrangements with repetition:
$number\ of\ arrangements = p!$
$= p \times (p-1) \times (p-2) \times (p-3)$ ...

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<td>

Example

How many three letter codes can be made from the letters in the word SPRING, if the letters cannot be repeated?

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Solution:

Number of ways $= 5 \times 4 \times 3$
$= 60$

</td>
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</table>

Section A:

Question 1

A survey was conducted among 100 boys and 60 girls to determine how many of them watched TV in the period during which examinations were written. Their responses are shown in the partially completed table below.

	WATCHED TV DURING EXAMINATIONS	DID NOT WATCH TV DURING EXAMINATIONS	TOTALS
Male	80	*a*	
Female	48	12	
Totals	*b*	32	160

1.1 Calculate the values of *a* and *b*. (2)

1.2 Are the events 'being a male' and 'did not watch TV during examinations' mutually exclusive? Give a reason for your answer. (2)

1.3 If a learner who participated in this survey is chosen at random, what is the probability that the learner:

1.3.1 Watched TV in the period during which the examinations were written? (2)

1.3.2 Is not a male and did not watch TV in the period during which examinations were written? (2)

Question 2

A tournament organiser conducted a survey among 150 members at a local sports club to find out whether they play tennis or not. The results are shown in the table below.

	PLAYING TENNIS	NOT PLAYING TENNIS
Male	50	30
Female	20	50

2.1 What is the probability that a member selected at random is:

(a) Female (2)

(b) Female and plays tennis (1)

2.2 Is playing tennis independent of gender? Motivate your answer with the necessary calculations. (3)

Question 3

The probability of events A and B occurring are denoted by P(A) and P(B) respectively.

For any two events A and B it is given that:

- $P(B') = 0{,}28$
- $P(B) = 3P(A)$
- $P(A \text{ or } B) = 0{,}96$

Are events A and B mutually exclusive? Justify your answer. (4)

Question 4 [6]

Each passenger on a certain Banana Airways flight chose exactly one beverage from tea, coffee or fruit juice. The results are shown in the table below.

	MALE	FEMALE	TOTAL
Tea	20	40	60
Coffee	***b***	***c***	80
Fruit juice	***d***	***e***	20
TOTAL	60	100	***a***

4.1 Write down the value of ***a.*** (1)

4.2 What is the probability that a randomly selected passenger is male? (2)

4.3 Given that the event of a passenger choosing coffee is independent of being a male, calculate the value of ***b.*** (4)

Question 5

For two events, A and B, it is given that:

P(A) = 0,2

P(B) = 0,63

P(A and B) = 0,126

Are the events, A and B, independent? Justify your answer with appropriate calculations. (3)

Question 6

Zebra High School offers only two sporting activities, namely rugby and hockey.

The following information is given:

- There are 600 learners in the school.
- 3 72 learners play hockey.
- 288 learners play rugby.
- 56 of the learners play NO sport.
- The number of learners that play both hockey and rugby is x.

6.1 Represent the given information in a Venn diagram, in terms of x. (3)

6.2 Calculate the value of x. (2)

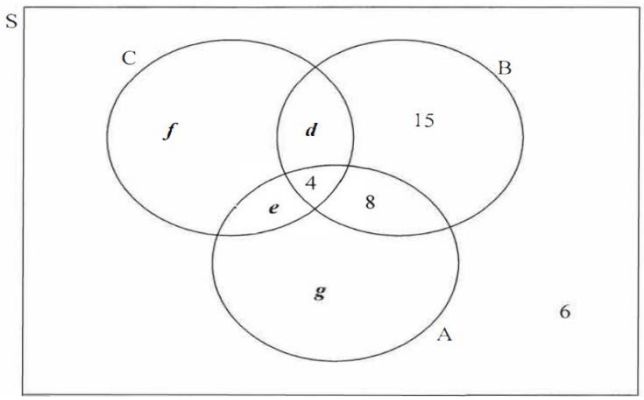
6.3 Are the events playing rugby and playing hockey mutually exclusive?
Justify your answer. (2)

Question 7

Research was conducted about driving under the influence of alcohol. Information obtained from traffic authorities in 54 countries on the methods that are used to measure alcohol levels in a person, are summarised below:

- 4 countries use all three methods (A, B and C).
- 12 countries use the alcohol content of breath (A) and blood-alcohol concentration (B).
- 9 countries use blood-alcohol concentration (B) and certificates issued by doctors (C).
- 8 countries use the alcohol content of breath (A) and certificates issued by doctors (C).
- 21 countries use the alcohol content of breath (A).
- 32 countries use blood-alcohol concentration (B).
- 20 countries use certificates issued by doctors (C).
- 6 countries use none of these methods.

Below is a partially completed Venn diagram representing the above information.



7.1 Use the given info1mation and the Venn diagram to determine the values of d, e, f and g. (4)

7.2 For a randomly selected country, calculate:

(a) P(A and B and C) (1)

(b) P(A or B or C) (1)

(c) P(only C) (1)

(d) P(that a country uses exactly two methods) (1)

Question 8

Events A and B are mutually exclusive. It is given that:

- P(B) = 2P(A)
- P(A or B) = 0,57

Calculate P(B). (3)

Question 9

Two identical bags are filled with balls. Bag A contains 3 pink and 2 yellow balls.

Bag B contains 5 pink and 4 yellow balls. It is equally likely that Bag A or Bag B is chosen. Each ball has an equal chance of being chosen from the bag. A bag is chosen at random and a ball is then chosen at random from the bag.

9.1 Represent the information by means of a tree diagram. Clearly indicate the probability associated with each branch of the tree diagram and write down all the outcomes. (4)

9.2 What is the probability that a yellow ball will be chosen from Bag A? (1)

9.3 What is the probability that a pink ball will be chosen? (3)

Question 10

Veli and Bongi are learners at the same school. Some days they arrive late at school.

The probability that neither Veli nor Bongi will arrive late on any day is 0,7.

10.1 Calculate the probability that at least one of the two learners will arrive late on a randomly selected day. (1)

10.2 The probability that Veli arrives late for school on a randomly selected day is 0,25, while the probability that both of them arrive late for school on that day is 0,15. Calculate the probability that Bongi will arrive late for school on that day. (3)

10.3 The principal suspects that the latecoming of the two learners is linked. The principal asks you to determine whether the events of Veli arriving late for school and Bongi arriving late for school are statistically independent or not. What will be your response to him? Show ALL calculations. (3)

Section B:

Question 1

Five boys and four girls go to the movies. They are all seated next to each other in the same row.

1.1 One boy and girl are a couple and want to sit next to each other at any end of the row of friends. In how many different ways can the entire group be seated? (3)

1.2 If all the friends are seated randomly, calculate the probability that all the girls are seated next to each other. (3)

Question 2

The letters of the word DECIMAL are randomly arranged into a new 'word', also consisting of seven letters. How many different arrangements are possible if:

2.1 Letters may be repeated (2)

2.2 Letters may not be repeated (2)

2.3 The arrangements must start with a vowel and end in a consonant and no repetition of letters is allowed. (4)

Question 3

Tulani High, has a sports awards ceremony. Tulani High has a basketball team consisting of 5 players and a volleyball team consisting of 6 players.

3.1 All the basketball players sit in a single row at the ceremony. There are no restrictions on who sits in which position. In how many different ways can they be seated? (1)

3.2 The decision is taken that the captain must sit in the first seat of the row. The two vice-captains have to be seated next to each other in any of the remaining seats. In how many different ways can the basketball players be seated now? (3)

3.3 After the interval, the basketball team and the volleyball team sit in the same row at the ceremony. Calculate the probability that the basketball players will sit together and the volleyball -players will sit together. Assume that seating positions are allocated randomly. Give your answer as a simplified fraction. (3)

Question 4

The digits 1 to 7 are used to create a four-digit code to enter a locked room. How many different codes are possible if the digits may not be repeated and the code must be an even number bigger than 5 000? (5)

Question 5

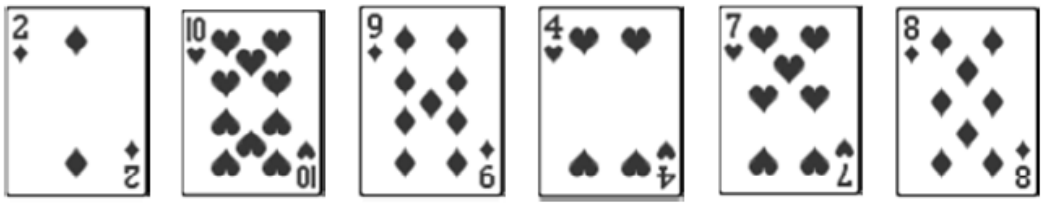
Consider the word M A T H S.

5.1 How many different 5-letter arrangements can be made using all the above letters? (2) (2)

5.2 Determine the probability that the letters S and T will always be the first two letters of the arrangements in QUESTION 5.1. (3)

Question 6

The cards below are placed from left to right in a row.



6.1 In how many different ways can these 6 cards be randomly arranged in a row? (2)

6.2 In how many different ways can these cards be arranged in a row if the diamonds and hearts are placed in alternating positions? (3)

6.3 If these cards are randomly arranged in a row, calculate the probability that ALL the hearts will be next to one another. (3)

Question 7

Cindy has the following books on a bookshelf:

- 4 Mathematics books
- 3 Ohysical Science books
- 2 Life Sciences books

7.1 Determine the number of different ways that all the books can be arranged. (2)

7. 2 Determine the number of different ways that the books can be arranged in order that the books in each learning area are next to each other. (3)

7.3 In how many different ways can all the books be arranged in order of descending height? (2)

Question 8

Consider the letters of the word: NUMERATOR.

8.1 How many 9 letter word-arrangements can be formed, if repitition of letters is allowed. (1)

8.2 How many 9 letter word-arrangements can be formed, if all 4 vowels are never together and repitition of letters is not allowed? (3)

8.3 An 8 letter word-arrangement is made from the NUMERATOR. All the vowels must be included in this word-arrangement and repitition of letters is not allowed. What is the probability that all odd-number spaces are accupied by vowels? (4)

Statistics

Summary: All content from gr 10 to 12 are assessed in the Gr 12 final examination.

1. **Summary of types of data:**

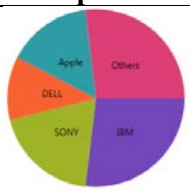
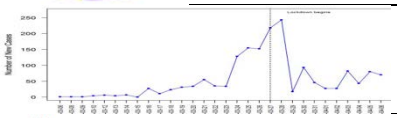
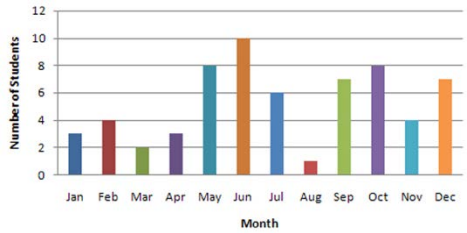
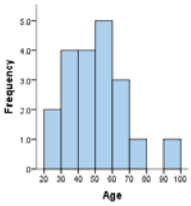
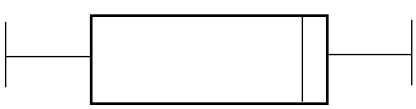
The table shows the difference between **univariate** and **bivariate data**

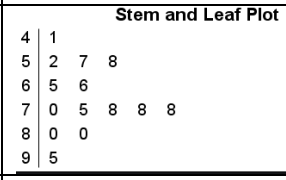
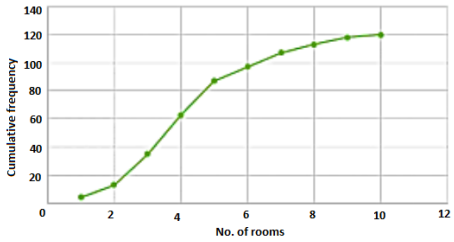
Univariate Data	Bivariate Data
Data that consists of a single variable **Example**: Mathematics test scores	Data that consists of two variables **Example**: time spent studying mathematics and the corresponding mathematics test marks.
Data is analysed by: • Determining the measures of central tendency- mean, mode, median. • Determining measures of dispersion-range, interquartile range, variance and standard deviation. • Drawing different graphs.	Data is analysed by: • Identifying the dependent and independent variables. • Determining if a relationship or correlation exist between the variables. • Determining the strength of the relationship or correlation.
Interpret questions like: how many learners have achieved a pass mark?	**Interpret** questions like: Describe the relationship/ correlation between the time spent studying and test scores.

Discrete data: Exact values that are countable whole numbers, for example the number of people that have recovered from the COVID-19 virus.

Continuous data: It can be values anywhere within a range of real number values, for example height, mass and time.

2. **Summary of types of graphs**

Type	Example	Explanatory notes
Pie chart Proportional relationships at a specific point in time		To convert data into degrees $\frac{frequency}{total\ outcomes} = 360°$
Line graph Trends/ changes over time.		Multiple trends can be compared
Bar graph Comparison of discrete non-numerical data. The heights of each bar represents the frequency.		Cannot be used to illustrate continuous data. Bar graphs are separated by distinct gaps between the bars
Histogram Trends in numerical continuous data. The area of the bars represents frequency.		Used only for continuous data with no gaps between bars. The bars can be of different widths, in keeping with interval size.
Box and whisker diagram Created by using a five number summary. Illustrates the spread of data. Can be used		Scale used must be accurate to extract an accurate five-point summary. minimum

to illustrate and discuss skewness		value, Q_1, Q_2, Q_3 and the maximum value.
Stem and leaf plot Used to summarise grouped data and simultaneously gives a *picture* of the data		The stems are the digits in the LHS column and the leaves are the digits in the RHS column.
Cumulative frequency curve / Ogive It is a graph that shows the information in a cumulative frequency table. You can draw an Ogive of ungrouped and grouped discrete data or grouped continuous data		The total of scores are called the cumulative frequency. It is calculated by adding the frequencies of all the previous scores. A smooth graph of cumulative frequencies.

3. Important symbols used in Statistics:

Σ	The Greek letter *sigma*, which means *sum of*, used to show that u must add all the values together
n	The number of scores (data items) in a data set.
$\bar{x}$	The mean/ average of all the scores in the data set.
σ	The standard deviation

4. Measuring Data

Central tendency:

The measures of central tendency are the three different averages i.e. the mean, median and mode.

Dispersion:

The measures of dispersion, i.e. the range, interquartile range, semi-IQR, variance and standard deviation are used to measure the spread and variability of the data.

	How to determine:	**Relevant Formulae**
Mean / Average	$mean = \frac{sum\ of\ all\ the\ scores}{number\ of\ scores}$	**Ungrouped data:** $\bar{x} = \frac{\Sigma x}{n}\ or\ \bar{x} = \frac{\Sigma fx}{n}$ **Grouped data** $\bar{x} = \frac{\Sigma(f \times interval\ midpoint}{n}$
Median: Middlemost scores	Arrange the scores in ascending order, if the number of the scores is: **Odd**, then the median is the score exactly in the middle. **Even**, add the two middle scores together and divide the result by 2	median$\left(\frac{n+1}{2}\right)$ use this formula for large samples. For grouped data the median can be estimated using an Ogive.
Mode	The score that occurs most often. More than one mode can exist. For grouped data use the modal interval	
Percentile	A percentile is a measure that tells us what % of the total frequency scored at, or below the measure. It divides data up into 100 equal parts.	To find the position of the k^{th} percentile which is a particular percentile: $P_k = \frac{k(n+1)}{100}$
Quartiles	Divide data up into 4 equal parts. The 1st quartile is the 25th percentile The 2nd quartile is the 50th percentile The 3rd quartile is the 75th percentile.	For large samples $Q_1 = \left(\frac{n+1}{4}\right) th\ score$ $Q_2 = \left(\frac{n+1}{2}\right) th\ score$

		$Q_3 = \left(\frac{3(n+1)}{4}\right) th\ score$
Range	The difference between the highest and lowest scores in a given data set.	Range is equal to highest score – lowest score
Interquartile Range (IQR)	The difference between the upper quartile and the lower quartile in a given set	IQR = $Q_3 - Q_1$
Semi - IQR	Half the difference between the upper and lower quartile in any given set	Semi-IQR $\frac{Q_3 - Q_1}{2}$

5. **Variance and Standard Deviation:** Two measures of dispersion, taking into account all of the data, which is linked to the mean.

The **Variance** is the mean of the sums of the squares of the deviations from the mean.
We find the variance by:

- Finding the mean: $\bar{x} = \frac{\Sigma x}{n}$
- Finding the deviation from the mean of each item of the data set:
  - **Deviation = data item (x) – mean ($\bar{x}$) = (x - $\bar{x}$)**
- Squaring each deviation: $\boldsymbol{(deviation)^2 = (x - \bar{x})^2}$
- Finding the sum of the squares of the deviation:
  - $\boldsymbol{\Sigma(deviation)^2 = \Sigma(x - \bar{x})^2}$
- Finding the mean of the squares of the deviations by dividing by the number of terms in the data set:
  - **Variance** = $\frac{\boldsymbol{\Sigma(deviation)^2}}{\boldsymbol{number\ of\ data\ items}} = \frac{\boldsymbol{\Sigma(x - \bar{x})^2}}{\boldsymbol{n}}$

The **Standard Deviation** is the square root of the variance:

$$\sqrt{\frac{\Sigma(x - \bar{x})^2}{n}}$$

- When data elements are closely/ tightly clustered together, the standard deviation and variance will be small, when they are spread apart, the standard deviation and the variance will be relatively large.
- A data set with more data items near to the mean will have less spread and a smaller standard deviation
- A data set with more data items far from the mean will have a greater spread and a larger standard deviation.

Example 1

a) Calculate the variance and standard deviation of the following two sets of data representing the number of runs scored by two cricketers over 10 matches.

Batsman A	40	45	51	52	62
Batsman B	15	28	44	78	85

b) Use the two standard deviations to compare the distribution of data in the two sets.

Solution 1

a)

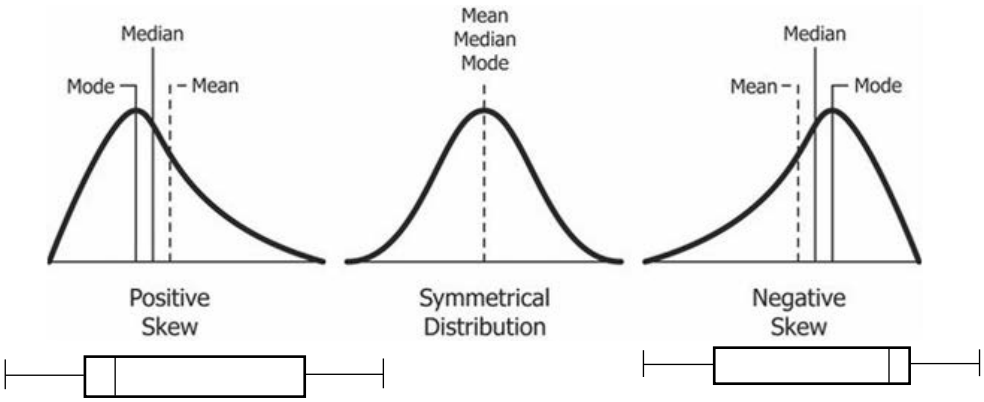
Batsman A	**Batsman B**
Variance = 54.8	Variance = 750.8
Standard Deviation = ≈ 7.	Standard Deviation = ≈ 27.4

b) The larger standard deviation with **Batsman B** indicates that the data items are generally much further from the mean than the data items with **Batsman A**.
This implies that the data items of **Batsman B** are more spread out than the data items of **Batsman A**.

In terms of cricket the difference in the standard deviations shows that **Batsman A** is possibly more predictable.

9. The distribution/ spread of Data

The distribution is the nature and shape of the spread of data. It can be either symmetric or skewed.



features of skewed data:

Skewness is the tendency for the values to be more frequently around the high or low ends of the x - axis

- With a positively skewed distribution, the tail on the right side is longer than the left side. (median $<$ mean)
  Most of the values tend to cluster toward the left side of the *x*-axis (i.e. the smaller values) with increasingly fewer values on the right side of the *x*-axis (i.e. the larger values)

- With a negatively skewed distribution, the tail on the left side is longer than the right side. (median $>$ mean)
  Most of the values tend to cluster toward the right side of the *x*-axis (i.e. the larger values) with increasingly fewer values on the left side of the *x*-axis (i.e. the smaller values.

10. Outliers

- An outlier is a data entry that is far removed from the other entries in a data set e.g. a data entry that is much smaller or larger than the rest of the data values.
- An outlier has an influence on the mean and the range of the data set, but has no influence on the mean or lower and upper quartiles.
- Any data item that is less than $Q_1 - 1{,}5 \times IQR$ or more than$Q_3 + 1{,}5 \times IQR$ is an outlier

Example:

Are any of the entries in the data set outliers?

1, 8, 12, 14, 14, 15, 17, 17, 19, 26, 32

First find the IQR

$$\text{IQR} = Q_3 - Q_1$$
$$19 - 12 = \mathbf{7}$$

Lower quartile (Q_1) $< Q_1 - 1{,}5 \times IQR$
$< 12 - 1{,}5 \times \mathbf{7}$
$< 1{,}5$
1 is an outlier

Upper quartile (Q_3) $> Q_3 + 1{,}5 \times IQR$
$> 19 + 1{,}5 \times \mathbf{7}$
$> 29{,}5$
32 is also an outlier

11. Scatter plots and types of Correlation

A scatter plot is a graph that helps you to see whether there is a correlation (relationship) between any set of two numeric data:

- It has two axes, one for each variable.
- Each ordered pair of values is plotted as a point on the graph.
- The x-coordinate is the independent variable.
- The y-coordinate is the dependent variable

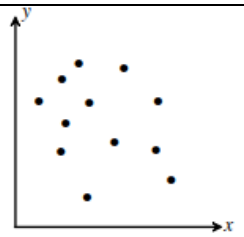
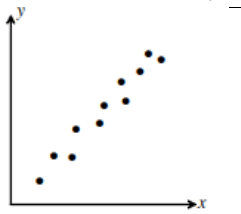
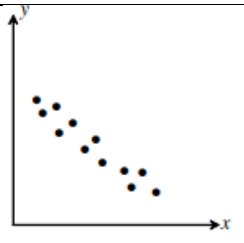
a) Types of Correlation

In analysing the scatter plot, you look at the way the points lie. Certain patterns tell you that correlations, (relationships) exist between the two variables.

When describing the correlation between two variables on a scatter plot, we should comment on:

- **The form**- whether it is linear, quadratic or exponential.
- **The direction**- whether it is negative or positive.
- **The strength** –whether it is strong, moderate or weak

The table indicates the different types of correlation:

No Correlation The points are scattered randomly over the graph, indicating no pattern between the two sets of data	
Strong positive linear correlation The points show a *band* that slopes upwards from bottom left to top right. As one variable increases, the other variable also increases.	
Strong negative linear correlation The points show a *band* that slopes downwards from top left to bottom right. As one variable increases, the other variable decreases.	

b) Correlation Coefficient (r)

It is the measure of strength and direction of the linear relationship between two variables.

The range of the correlation coefficient is between -1 and 1

- If x and y have a strong positive correlation, r will be close to 1
- If x and y have a strong negative correlation, r will be close to -1
- If there is no linear correlation or there is a very weak linear correlation r will be close to 0

12. Line of best fit and regression line

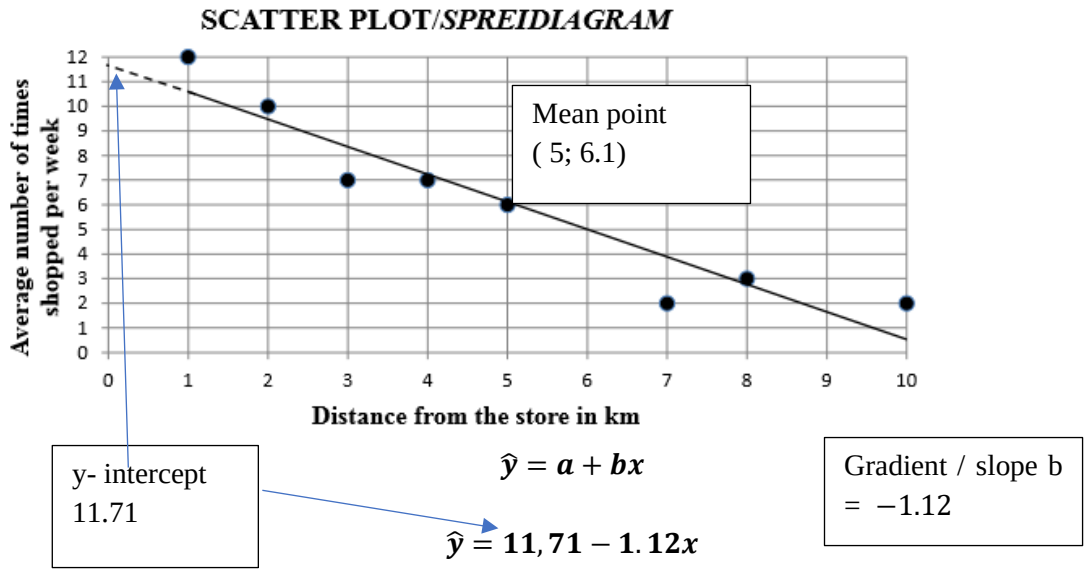
The strength of the linear relationship between the two variables in a scatterplot depends on how close the data points are to the line of best fit. The closer the points are to this line, the stronger the relationship.

Moving from drawing lines of best fit intuitively to finding the equation of the actual line of best fit for data represented on a scatter plot.

The method involves determining the gradient and y – intercept of the line of best fit by using a calculator.

The equation of the regression line is given by $\hat{y} = a + bx$ where a represents the y – intercept of the line and b the gradient or slope.

The line of best fit passes through the mean point $(\bar{x}; \bar{y})$ (5; 6.1)



Using least squares regression lines to make predictions

- When a value for one of the variables that was not originally in the data is found, you can make a prediction.
- The required value can be read off from the scatter plot or by using the equation of the regression line.
- Predictions can be made through the process of interpolation and extrapolation
- **Interpolation**- new data item within the range of data items
- **Extrapolation**- estimating new data values beyond a set of known data values.

Section A:
STATISTICS

QUESTION 1 **DBE-May /June 2019 (Question 1)**

Each child in a group of four-year-old children was given the same puzzle to complete. The time taken (in minutes) by each child to complete the puzzle is shown in the table below.

TIME TAKEN (t) (IN MINUTES)	**NUMBER OF CHILDREN**
$2 < t \leq 6$	2
$6 < t \leq 10$	10
$10 < t \leq 14$	9
$14 < t \leq 18$	7
$18 < t \leq 22$	8
$22 < t \leq 26$	7
$26 < t \leq 30$	2

1.1 How many children completed the puzzle? (1)
1.2 Calculate the estimated mean time taken to complete the puzzle. (2)
1.3 Complete the cumulative frequency column in the table (2)
1.4 Draw a cumulative frequency graph (ogive) to represent the data (3)
1.5 Use the graph to determine the median time taken to complete the puzzle. (2)
[10]

QUESTION 2 **DBE-May /June 2019 (Question 2)**

Learners who scored a mark below 50% in a Mathematics test were selected to use a computer-based programme as part of an intervention strategy. On completing the programme, these learners wrote a second test to determine the effectiveness of the intervention strategy. The mark (as a percentage) scored by 15 of these learners in both tests is given in the table below.

LEARNER	**L1**	**L2**	**L3**	**L4**	**L5**	**L6**	**L7**	**L8**	**L9**	**L 10**	**L 11**	**L 12**	**L 13**	**L 14**	**L 15**
TEST 1 (%)	10	18	23	24	27	34	34	36	37	39	40	44	45	48	49
TEST 2 (%)	33	21	32	20	58	43	49	48	41	55	50	45	62	68	60

2.1 Determine the equation of the least squares regression line. (3)
2.2 A learner's mark in the first test was 15 out of a maximum of 50 marks.
2.2.1 Write down the learner's mark for this test as a percentage. (1)
2.2.2 Predict the learner's mark for the second test. Give your answer to the nearest integer. (2)

2.3 For the 15 learners above, the mean mark of the second test is 45,67% and the standard deviation is 13,88%. The teacher discovered that he forgot to add the marks of the last question to the total mark of each of these learners. All the learners scored full marks in the last question. When the marks of the last question are added, the new mean mark is 50,67%.
2.3.1 What is the standard deviation after the marks for the last question are added to each learner's total? (2)
2.3.2 What is the total mark of the last question? (2)

Question 3 **DBE Nov 2019 (Question 1)**

The table below shows the monthly income (in rands) of 6 different people and the amount (in rands) that each person spends on the monthly repayment of a motor vehicle.

Monthly income(in rands)	9 000	13 500	15 000	16 500	17 000	20 000
Monthly repayments (in rands)	2 000	3 000	3 500	5 200	5 500	6 000

3.1 Determine the equation of the least squares regression line for the data (3)

3.2 If a person earns R14 000 per month, predict the monthly repayment that the person could make towards a motor vehicle. (2)

3.3 Determine the correlation coefficient between the monthly income and the monthly repayment of a motor vehicle. (1)

3.4 A person who earns R18 000 per month has to decide whether to spend R 9000 as a monthly repayment of a motor vehicle, or not. If the above information is a true representation of the population data, which of the following would the person most likely decide on:

A Spend R9 000per month because there is a very strong positive correlation between the amount earned and the monthly repayment.

B Not to spend the R9 000 per month because there is a very weak positive correlation between the amount earned and the monthly repayment.

C Spend R9 000 per month because the point (18 000; 9 000) lies very near to the least squares regression line.

D Not to spend R9 000 per month because the point (18 000; 9 000) lies very far to the least squares regression line.

(2)

Question 4 **DBE Nov 2019 (Question 2)**

A survey was conducted among 100 people about the amount that they paid on a monthly basis for their cellphone contracts. The person carrying out the survey calculated the estimated mean to be R309 per month. Unfortunately, he lost some of the data thereafter. The partial results of the survey are shown in the frequency table below:

AMOUNT PAID (IN RANDS)	**FREQUENCY**
$0 < x \leq 100$	7
$100 < x \leq 200$	12
$200 < x \leq 300$	$\boldsymbol{a}$
$300 < x \leq 400$	35
$400 < x \leq 500$	$\boldsymbol{b}$
$500 < x \leq 600$	6

4.1 How many people paid R200 or less on their monthly cell phone contracts? (1)

4.2 Use the information above to show that $a = 24 \ \ and \ b = 16$ (5)

4.3 Write down the modal class for the data. (1)

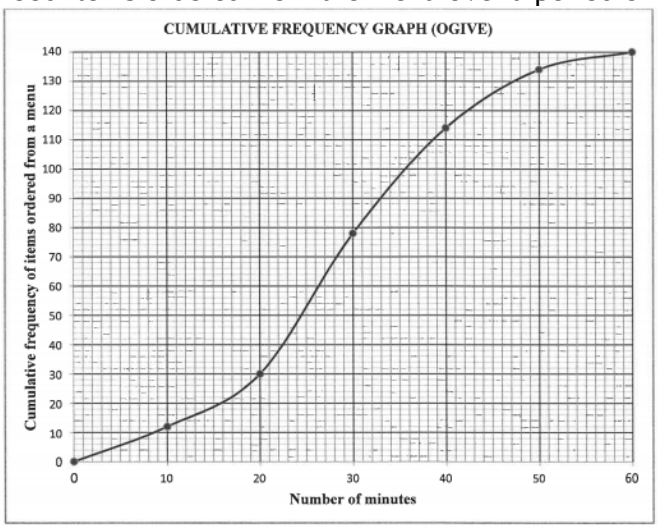
4.4 Draw an ogive (cumulative frequency graph) to represent the data. (4)

4.5 Determine how many people paid more than R420 per month for their cell phone contracts. (2)

[13]

Question 5 **DBE Nov 2018 (Question 1)**

The cumulative frequency graph (ogive) drawn below shows the total number of food items ordered from the menu over a period of 1 hour.



5.1 Write down the total number of food items ordered from the menu during this hour. (1)

5.2 Write down the modal class of the data. (1)

5.3 How long did it take to order the first 30 food items? (1)

5.4 How many food items were ordered in the last 15 minutes? (2)

5.5 Determine the 75^{th} percentile for the data. (2)

5.6 Calculate the interquartile range of the data. (2)
[9]

Question 6 **DBE Nov 2018 (Question 2)**

Reggie works part-time as a waiter at a local restaurant. The amount of money (in rands) he made in tips over a 15-day period is given below.

35	70	75	80	80
90	100	100	105	105
110	1110	115	120	125

6.1.1 Calculate

a) The mean of the data (2)

b) The standard deviation of the mean. (2)

6.1.2. Mary also works part-time as a waitress at the same restaurant. Over the same 15day period, Mary collected the same amount in tips as Reggie, but her standard deviation was R14. Using the available information, comment on the:

a) Total amount in tips that they each collected over the 15day period. (1)

b) Variation that each of them received in daily tips over this period. (1)
[6]

Question 7 **DBE May/June 2018 (Question 1)**

The monthly profit (in thousands of rands) made by a company in a year is given in the table below.

110	112	156	164	167	169
171	176	192	228	278	360

7.1 Calculate the:

7.1.1 Mean profit for the year. (3)

7.1.2. Median profit for the year. (1)

7.2 Draw a box and whisker diagram to represent the data (2)

7.3 Hence,determine the interquartile range of the data. (1)

7.4 Comment on the skewness in the distribution of the data. (1)

7.5 For the given data:

7.5.1 Calculate the standard deviation (1)

7.5.2 Determine the number of months in which the profit was less than one standard deviation below the mean. (2)

[11]

Question 8 **DBE May/June 2018 (Question 2)**

It is said that the number of times that a cricket chirps in a minute gives a very good indication of the air temperature(in $°C$). The table shows the information recorded during an observation study.

Chirps per minute	Air temp in $°C$
32	8
40	10
52	12
76	15
92	17
112	20
128	25
180	28
184	30
200	35

8.1 Represent the data above on a scatter plot (3)

8.2 Explain why the claim, “gives a very good indication”, is TRUE. (1)

8.3 Determine the equation of the least squares regression line of the data. (3)

8.4 Predict the air temperature (in $°C$) if a cricket chirps 80 times a minute (2

[9]

Question 9 **DBE Feb/March 2018 (Question 1)**

An organisation decided that it would set up blood donor clinics at various colleges. Students would donate blood over a period of 10 days. The number of units of blood donated per day by students of college X is shown in the table below.

Days	1	2	3	4	5	6	7	8	9	10
Units of blood	45	59	65	73	79	82	91	99	101	106

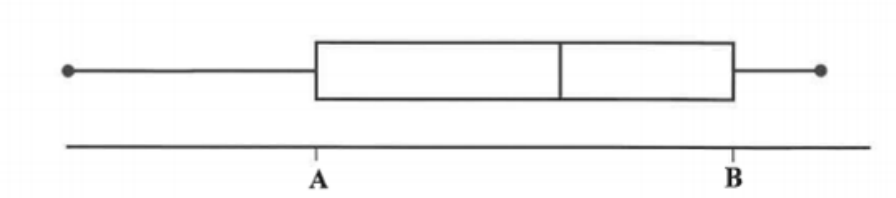
9.1 Calculate:

9.1.1 The mean of the units of blood donated per day over a period of 10 days (2)

9.1.2 The standard deviation. (2)

9.1.3 How many days is the number of units of blood donated at College X outside one standard deviation from the mean? (3)

9.2 The number of units of blood donated by students of College X is represented in the box and whisker diagram below.



9.2.1 Describe the skewness of the data (1)

9.2.2 Write down the valuesof A and B, the lowe quartile and the upper quartile Respectively, of the data set. (2)

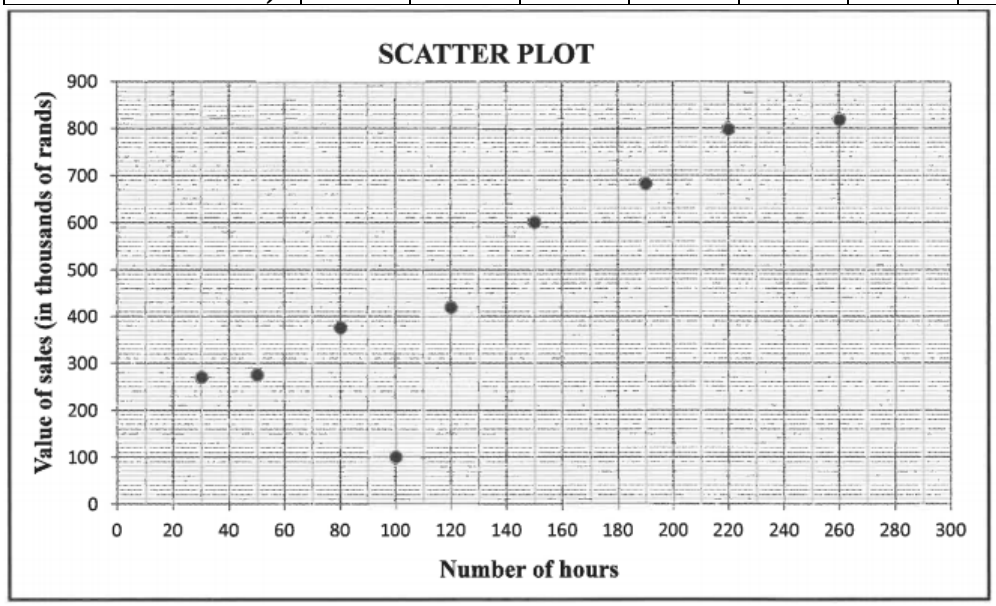
9.3 It was discovered that there was an error in counting the number of units of blood donated by college X each day. The correct mean of the data is 95 units of blood. How many units of blood were **not** counted over the ten days? (1)

[11]

Question 10 **DBE Feb/March 2018 (Question 2)**

The table below shows the number of hours that a sales representative of a company spent with each of his nine clients in one year and the value of the sales (in thousands of rands) for that client.

Number of hours	30	50	80	100	120	150	190	220	260
Value of sales(in thousand of rands)	270	275	376	100	420	602	684	800	820



10.1 Identify an outlier in the data above (1)

10.2 Calculate the equation of the least squares regression line of the data (3)

10.3 The sales representative forgot to record the sales of one of his clients, predict the value of this client's sales (in thousands of rands) if he spends 240 hours with him during a year. (2)

10.4 What is the expected increase in sales for each additional hour spent with a client? (2)

[8]

Trigonometry

Summary:

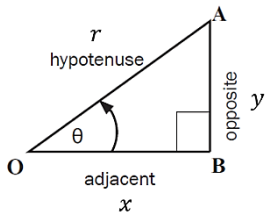
1. CAPS EXTRACTION INDICATING PROGRESSION FROM GRADES 10-12

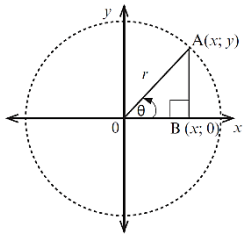
<table>
<tr><th>Grade 10-CAPS</th><th>Grade 11-CAPS</th></tr>
<tr><td rowspan="2">1. Define the trigonometric ratios sin θ, cos θ and tan θ, using right-angled triangles.
2. Extend the definitions of sin θ, cos θ and tan θ for $0^0 \le \theta \le 360^0$.
3. Define the reciprocals of the trigonometric ratios cosec θ, sec θ and cot θ, using right-angled triangles (these three reciprocals should be examined in grade 10 only).
4. Derive values of the trigonometric ratios for the special cases (without using a calculator)
$\theta \in \{0^0; 30^0; 45^0; 60^0; 90^0\}$.
5. Solve two-dimensional problems involving right-angled triangles.
6. Solve simple trigonometric equations for angles between 0^0 and 90^0.
7. Use diagrams to determine the numerical values of ratios for angles from 0^0 to 360^0.</td><td>(a) Derive and use the identities:
$\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sin^2\theta + \sin^2\theta = 1$.
(b) Derive the reduction formulae.
(c) Determine the general solution and / or specific solutions of trigonometric equations.
(d) Establish the sine, cosine and area rules.</td></tr>
<tr><td>Grade 12-CAPS</td></tr>
<tr><td></td><td>Proof and use of the compound angle and double angle identities
$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$;
$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$;
$\sin 2\alpha = 2\sin\alpha\cos\alpha$;
$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$;
$\cos 2\alpha = 2\cos^2\alpha - 1$; and
$\cos 2\alpha = 1 - \sin^2\alpha$.</td></tr>
<tr><td>TRIG GRAPHS Gr 10 & 11</td><td></td></tr>
<tr><td>Point by point plotting of basic graphs defined by $y = \sin\theta$, $y = \cos\theta$ and $y = \sin\theta$ for $\theta \in [-360°; 360°]$
Investigate the effect of the parameter k on the graphs of the functions defined by $y = \sin(kx)$, $y = \cos(kx)$ and $y = \tan(kx)$.
Investigate the effect of the parameter p on the graphs of the functions defined by $y = \sin(x + p)$, $y = \cos(x + p)$ and $y = \tan(x + p)$.
Draw sketch graphs defined by:
$y = a \sin k(x+p)$,
$y = a \cos k(x+p)$ and
$y = a \tan k(x+p)$ at most two parameters at a time.</td><td></td></tr>
</table>

2. DEFINITIONS; IDENTITIES AND FORMULAS

Note that some of the images used in the summary below is from "Mind The Gap".

Definitions





- $\boldsymbol{sin\theta = \frac{opp}{hyp} = \frac{y}{r}}$ $\quad$ $\boldsymbol{cosec\theta = \frac{hyp}{opp} = \frac{r}{y}}$
- $\boldsymbol{cos\theta = \frac{adj}{hyp} = \frac{x}{r}}$ $\quad$ $\boldsymbol{sec\theta = \frac{hyp}{adj} = \frac{r}{x}}$
- $\boldsymbol{tan\theta = \frac{opp}{adj} = \frac{y}{x}}$ $\quad$ $\boldsymbol{cot\theta = \frac{adj}{opp} = \frac{y}{x}}$

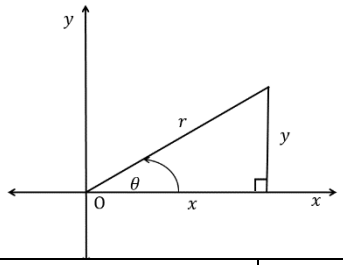
Identities

- $tan\theta = \frac{sin\theta}{cos\theta}$
- $sin^2\theta + cos^2\theta = 1$

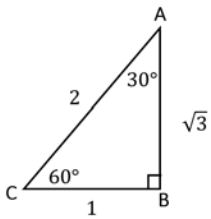
$1 - sin^2\theta = cos^2\theta \quad or \quad 1 - cos^2\theta = sin^2\theta$

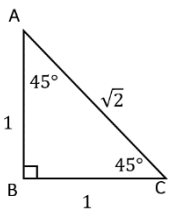
PROOF OF IDENTITIES

Proof of the identities are examinable.



Special angles





Proof:

$$\frac{sin\theta}{cos\theta}$$
$$= \frac{y}{r} \div \frac{x}{r}$$
$$= \frac{y}{r} \times \frac{r}{x}$$
$$= \frac{y}{x}$$
$$= tan\theta$$

Proof:

$$sin^2\theta + cos^2\theta$$
$$= \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2$$
$$= \frac{y^2}{r^2} + \frac{x^2}{r^2}$$
$$= \frac{y^2 + x^2}{r^2}$$
$$= \frac{r^2}{r^2}$$
$$= 1$$

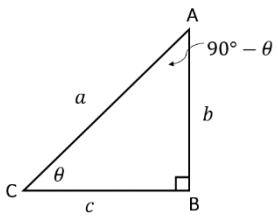
Reduction formulas

$sin(180° - \theta) = sin\theta$	$sin(180° + \theta) = -sin\theta$	$sin(360° - \theta) = -sin\theta$
$cos(180° - \theta) = -cos\theta$	$cos(180° + \theta) = -cos\theta$	$cos(360° - \theta) = cos\theta$
$tan(180° - \theta) = -tan\theta$	$tan(180° + \theta) = tan\theta$	$tan(360° - \theta) = -tan\theta$

Neagtive angles

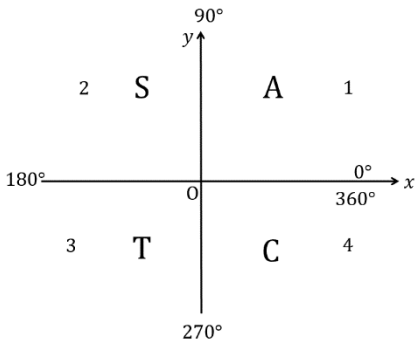
$sin(-\theta) = -sin\theta$	$cos(-\theta) = cos\theta$	$tan(-\theta) = -tan\theta$

Co-functions



$cos(90° - \theta) = sin\theta$
$cos(90° + \theta) = -sin\theta$
$sin(90° - \theta) = cos\theta$
$sin(90° + \theta) = cos\theta$

CAST-diagram



Compound and Double angles

- $\sin(\alpha+\beta) = sin\alpha cos\beta + cos\alpha sin\beta$
- $\sin(\alpha-\beta) = sin\alpha cos\beta - cos\alpha sin\beta$
- $\cos(\alpha+\beta) = cos\alpha cos\beta - sin\alpha sin\beta$
- $\cos(\alpha-\beta) = cos\alpha cos\beta + sin\alpha sin\beta$

- $sin2\alpha = 2sin\alpha cos\alpha$
- $cos2\alpha = cos^2\alpha - sin^2\alpha$
- $= 2cos^2\alpha - 1$
- $= 1 - 2sin^2\alpha$

Proof:

- $\cos(\alpha+\beta)$
  $= \cos[\alpha-(-\beta)]$
  $= cos\alpha\cos(-\beta) + sin\alpha\sin(-\beta)$
  $= cos\alpha cos\beta - sin\alpha sin\beta$

Proof:

- $\sin(\alpha+\beta)$
  $= \cos[90° - (\alpha+\beta)] = \cos[90° - \alpha - \beta]$
  $= \cos[(90° - \alpha) - \beta]$
  $= \cos(90° - \alpha).cos\beta + \sin(90° - \alpha).sin\beta$
  $= sin\alpha cos\beta + cos\alpha sin\beta$

Proof

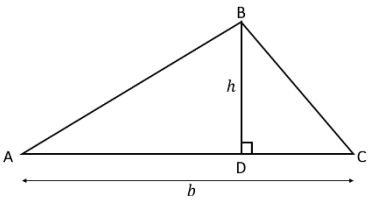
- $\sin(\alpha-\beta)$
  $= \cos[90° - (\alpha-\beta)] = \cos[90° - \alpha + \beta]$
  $= \cos[(90° - \alpha) + \beta]$
  $= \cos(90° - \alpha).cos\beta - \sin(90° - \alpha).sin\beta$
  $= sin\alpha cos\beta - cos\alpha sin\beta$

Undefined

- $\frac{any\ number}{0}$ is undefined
  Therefore if the denominator of an identity $= 0$, then the identity is undefined.
- $y = tanx$ is undefined forr certain values of x.
  $(x = 90° + 180k\ ; k \in Z)$
  Therfore if a tan function is in an identify is undefined where the tan function is undefined.

Area rule : $\boldsymbol{A = \frac{1}{2}absinC}$

If $\hat{A}$ *is acute*

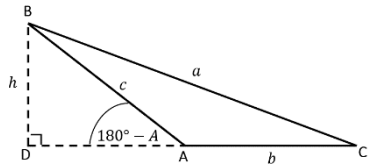


Area of ΔABC= $\frac{1}{2}bh$ ……(1)
But $sinA = \frac{h}{c}$ $\therefore h = csinA$

Substitution into (1)
Area of ΔABC= $\frac{1}{2}bcsinA$

Similarly it can be shown that
Area of ΔABC= $\frac{1}{2}absinC$
Area of ΔABC= $\frac{1}{2}acsinB$

If $\hat{A}$ *is obtuse*



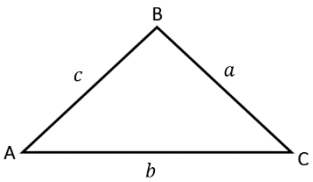
Area of ΔABC= $\frac{1}{2}bh$ ……(1)
But $\sin(180° - A) = \frac{h}{c}$
$\therefore h = csinA$

Substitution into (1)
Area of ΔABC= $\frac{1}{2}bcsinA$

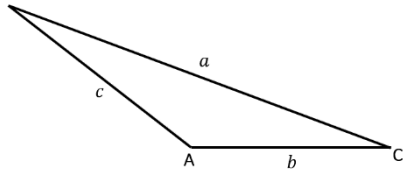
Similarly it can be shown that
Area of ΔABC= $\frac{1}{2}absinC$
Area of ΔABC= $\frac{1}{2}acsinB$

Sine rule: $\boldsymbol{\frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c}}$

If $\hat{A}$ *is acute*



If $\hat{A}$ *is obtuse*



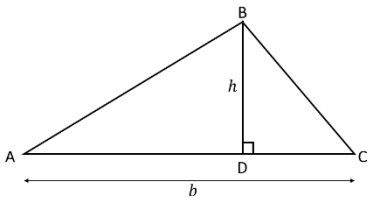
Using the area rule for ΔABC:

$$\frac{1}{2}bcsinA = \frac{1}{2}absinC = \frac{1}{2}acsinB$$

Dividing each by $\frac{1}{2}abc$ results: $\frac{sinA}{a} = \frac{sinC}{c} = \frac{sinB}{b}$

Cosine -rule: $\boldsymbol{a^2 = b^2 + c^2 - 2bc\ cosA}$

If $\hat{A}$ *is acute*



In ΔBDC: $a^2 = BD^2 + CD^2$ (Pythagoras Theorem)

$= BD^2 + (b - AD)^2$

$= BD^2 + b^2 - 2bAD + AD^2$

But $BD^2 + AD^2 = c^2$ (Pythagoras Theorem)

Thus $a^2 = b^2 + c^2 - 2bAD$ ... ... (1)

In ΔABD: $cosA = \frac{AD}{c}$ $\therefore AD = c\ cosA$ ... ... (2)

Substituting (2) into (1)

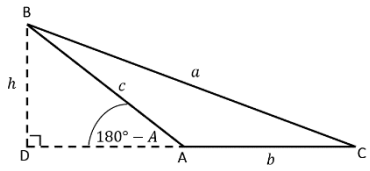
$\therefore a^2 = b^2 + c^2 - 2bc\ cosA$

Similarly it can be shown that:

$b^2 = a^2 + c^2 - 2ac\ cosB$ and

$c^2 = a^2 + b^2 - 2ab\ cosC$

If $\hat{A}$ *is obtuse*



In ΔBDC: $a^2 = BD^2 + CD^2$ (Pythagoras Theorem)

$= BD^2 + (b + AD)^2$

$= BD^2 + b^2 + 2bAD + AD^2$

But $BD^2 + AD^2 = c^2$ (Pythagoras Theorem)

Thus $a^2 = b^2 + c^2 + 2bAD$ ... ... (1)

In ΔABD: $\cos(180° - A) = \frac{AD}{c}$

$\therefore AD = c\ cosA$ ... ... (2)

Substituting (2) into (1)

$\therefore a^2 = b^2 + c^2 - 2bc\ cosA$

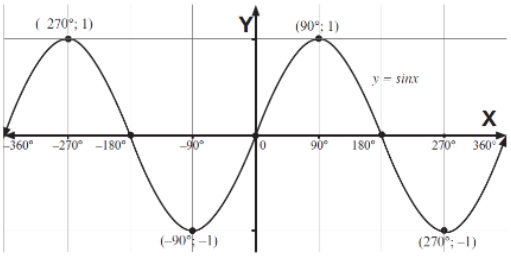
Similarly it can be shown that:

$b^2 = a^2 + c^2 - 2ac\ cosB$ and

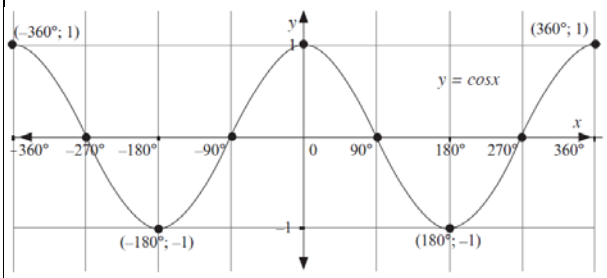
$c^2 = a^2 + b^2 - 2ab\ cosC$

Trig graphs

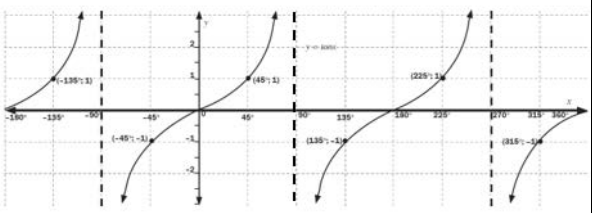
- $y = sinx$



- $y = cosx$

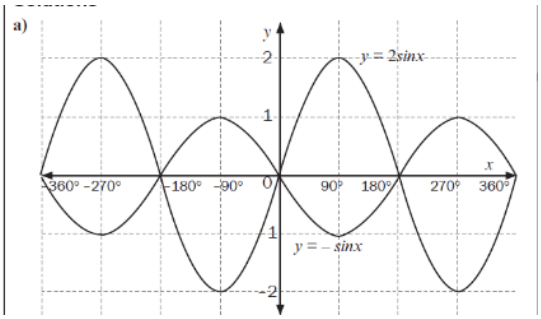


- $y = tanx$

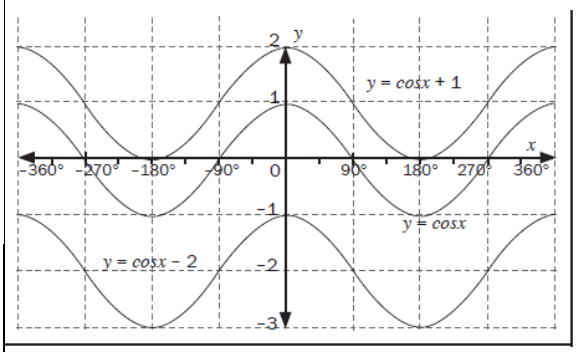


Asymptotes at $x = 90° + 180°k\ ;\ k \in Z$

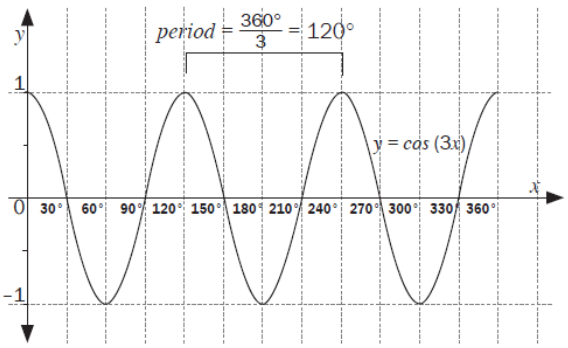
- $y = 2sinx$ → *amplitude change from 1 to 2*



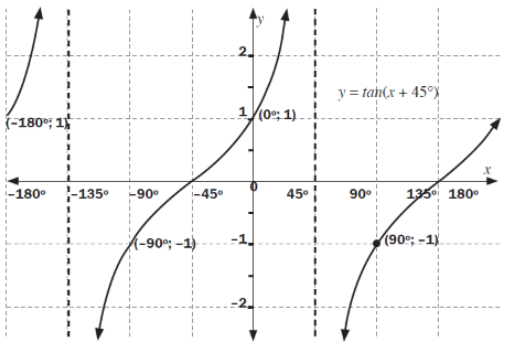
- $y = cosx + 1$ and $y = cosx - 2$
- *Graph moves 1 up / graph moves 2 down*



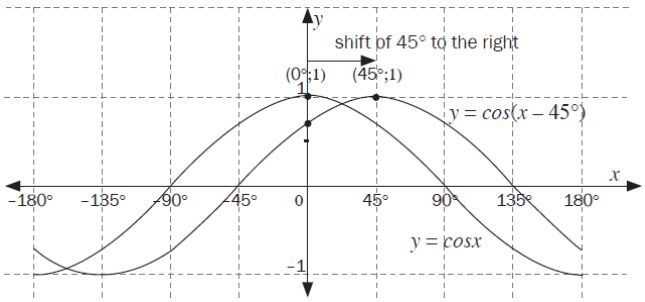
- $y = cos3x$ → *period change from 360° to* $\frac{360°}{3}$



- $y = tan\,(x + 45°)$ → *Graph moves 45° left*



- $y = \cos\,(x - 45°)$ → *Graph moves 45° right*



Section A:

NOTE:

- Questions in this revision were compiled from past exam National papers for GR 12.
- Some questions have been altered to fit the content covered in the FET lessons for Gr 12.

SECTION A (Routine questions)

QUESTION 1

(GR12 NSC NOV 2019)

1.1 1.1.1 Simplify the following expression to ONE trigonometric term:

$$\frac{\sin x}{\cos x.\tan x}+\sin(180°+x)\cos(90°-x)$$

1.1.2 Without using a calculator, determine the value of:

$$\frac{\sin^2 35°-\cos^2 35°}{4\sin 10°\cos 10°}$$

(GR12 NSC NOV 2017)

1.2 Given: $\dfrac{\sin(A-360°).\cos(90°+A)}{\cos(90°-A).\tan(-A)}$

Simplify the expression to a single trigonometric ratio.

(GR12 NSC NOV 2017)

1.3 Prove:

1.3.1 $\sin(A+B)-\sin(A-B)=2\cos A.\sin B$

1.3.2 Without using a calculator, that $\sin 17°-\sin 43°=\sin 17°$

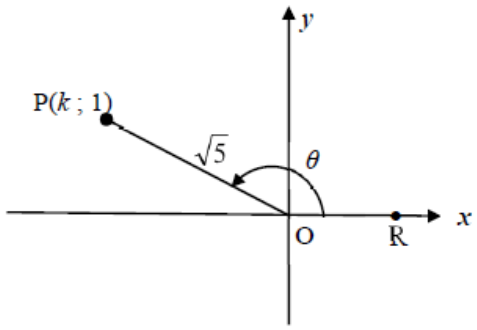
(GR12 NSC March 2018)

1.4 Simplify: $\sin(180°-x).\cos(-x)+\cos(90°+x).\cos(x-180°)$ to a single trigonometric ratio.

1.5 Determine the value of $\sin 3x.\cos y+\cos 3x.\sin y$ if $3x+y=270°$

QUESTION 2 (GR12 NSC NOV 2018)-Q5

2.1 In the diagram $P(k;1)$ is a point in the 2nd quadrant and is $\sqrt{5}$ units from the origin. R is a point on the positive $x-$ axis and obtuse $R\hat{O}P=\theta$



2.1.1 Calculate the value of k.

2.1.2 Without using a calculator, calculate the value of:

a) $\tan\theta$

b) $\cos(180°+\theta)$

c) $\sin(\theta+60°)$ in the form $\frac{a+b}{\sqrt{20}}$

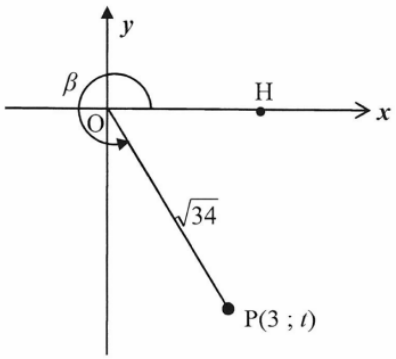
(GR12 NSC NOV 2017)-Q5

2.2 In the diagram, P$(3;t)$ is a point in the Cartesian plane. OP$=\sqrt{34}$ and H$\widehat{O}$P $=\beta$ is a reflex angle. Without using a calculator, determine the value of:

2.2.1. t

2.2.2. $\tan\beta$

2.2.3. $\cos 2\beta$



(GR12 NSC March 2018)-Q5

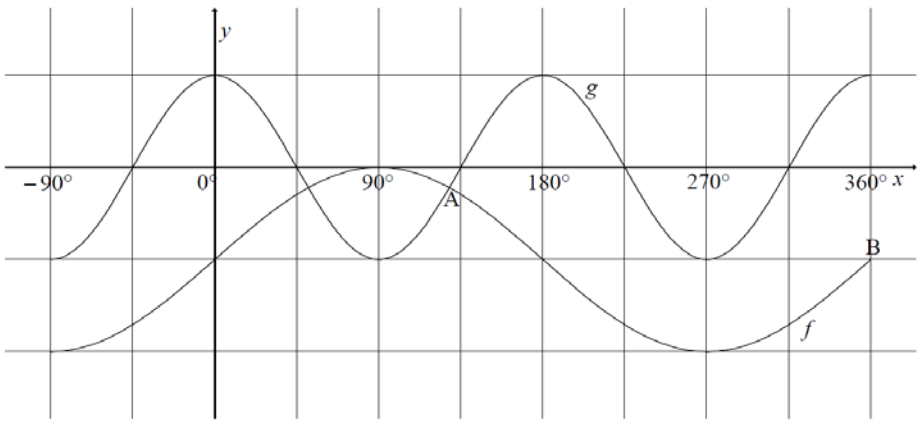
2.3 If $\cos 2\theta = -\frac{5}{6}$, where $2\theta \epsilon [180°;270°]$, calculate **without using a calculator**, the values in simplest form of

2.3.1 $\sin 2\theta$

2.3.2 $\sin^2\theta$

QUESTION 3 (GR 12 NSC November 2019)

3.1 In the diagram, the graphs of $f(x) = \sin x - 1$ and $g(x) = \cos 2x$ are drawn for the interval $x \in [-90° ; 360°]$. Graphs f and g intersect at A. B $(360° ; -1)$ is a point on f.

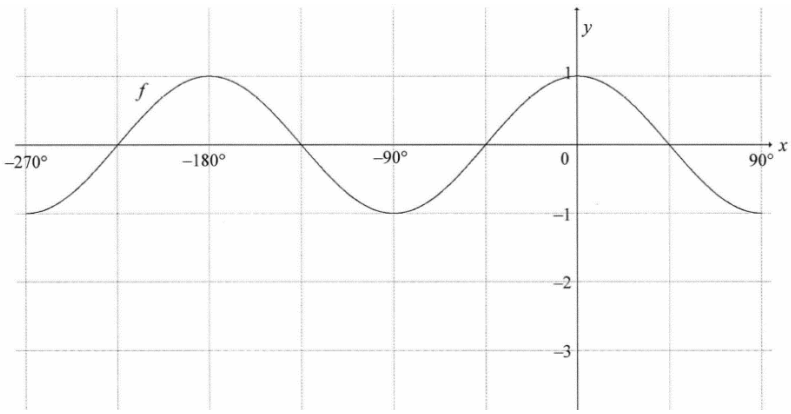


3.1.1 Write down the range of f.

3.1.2 Write down the values of x in the interval $x \epsilon [-90°; 360°]$ for which graph f is decreasing.

(GR 12 NSC November 2017)

3.2 In the diagram, the graph of $f(x) = \cos 2x$ is drawn for the interval $x \in [-270° ; 90°]$.



3.2.1 Draw the graph of $g(x) = 2\sin x - 1$ for the interval $x \in [-270° ; 90°]$ on the grid given. Show ALL intercepts with the axes, as well as the turning points.

3.2.2 Let A be a point of intersection of the graphs f and g. Show that the x −coordinate of A satisfies the equation $\sin x = \frac{-1+\sqrt{5}}{2}$.

3.2.3 Hence, calculate the coordinates of the points of intersection of graphs of f and g for the interval $x \in [-270° ; 90°]$.

QUESTION 4

4.1 Determine the general solution of the following equation:

4.1.1 $\sin 2x + \cos x = 0$

4.1.2 $\cos 2x - 5\cos x - 2 = 0$

(GR12 NSC March 2018)

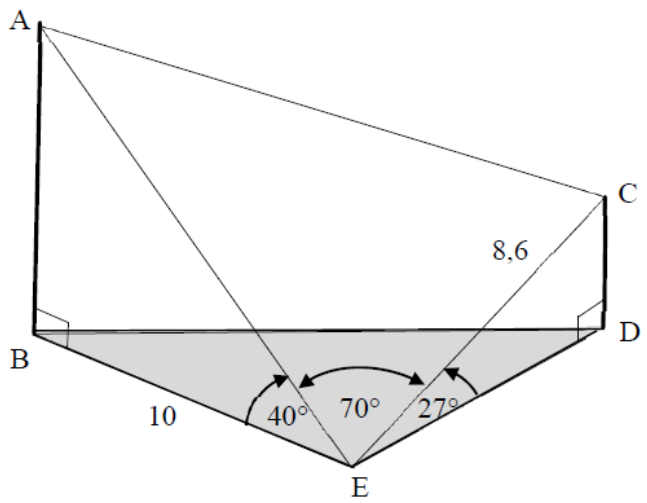
4.2 Given: $2\cos x = 3\tan x$

4.2.1 Show that the equation can be written as $2\sin^2 x + 3\sin x - 2 = 0$

4.2.2 Determine the general solution of x if $2\cos x = 3\tan x$.

QUESTION 5 (NSC March 2017)-Q7

In the diagram, B, E and D are points in the same horizontal plane. AB and CD are vertical poles. Steel cabels AE and CE anchor the poles at E. Another steel cable connects A and C. CE $= 8{,}6m$, BE$= 10m$, $A\hat{E}B = 40°$, $A\hat{E}C = 70°$ and $C\hat{E}D = 27°$.



Calculate the:

5.1 Height of pole CD.

5.2 Length of cable AE.

5.3 Length of cable AC.

Section **B:** **(Complex questions)**

QUESTION 1

(GR12 NSC March 2016)

1.1 1.1.1 Simplify $\frac{4\sin x\cos x}{2\sin^2 x-1}$ to a single trigonometric ratio.

1.1.2 Hence, calculate (WITHOUT a calculator) the value of the following: (leave your answer in simplest surd form.)

$$\frac{4\sin 15°\cos 15°}{2\sin^2 15°-1}$$

(GR12 NSC NOV 2018)

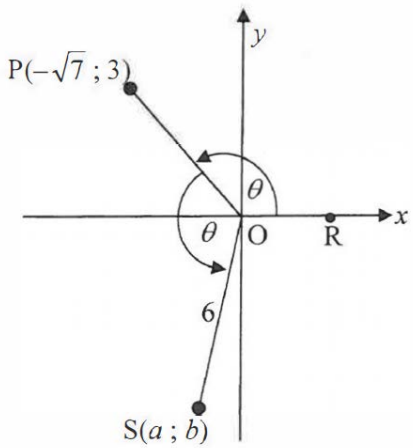
1.2 Prove the following identity: $\frac{\cos x+\sin x}{\cos x-\sin x}-\frac{\cos x-\sin x}{\cos x+\sin x}=2\tan x$

QUESTION 2

(GR12 NSC March 2016)

2.1 $P(-\sqrt{7};3)$ and $S(a\,;b)$ are points on the Cartesian plane, as shown in the diagram below.

$P\hat{O}R = P\hat{O}S = \theta$ and OS= 6



Determine, WITHOUT using a calculator, the value of:

2.1.1 $\tan\theta$

2.1.2 $\sin(-\theta)$

2.1.3 a

(GR12 NSC NOV 2019)

2.2 Given: $\cos 26° = m$

Without using a calculator, determine $2\sin^2 77°$ in terms of m.

(GR12 NSC NOV 2015)

2.3 Given that $\sin 23° = \sqrt{k}$, determine, in its simplest form, the value of each of the following in terms of k, WITHOUT using a calculator.

2.3.1 $\sin 203°$

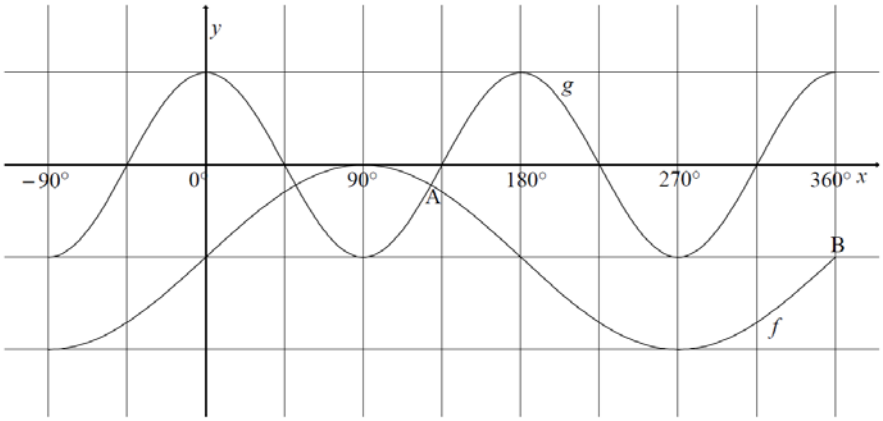
2.3.2 $\cos 23°$

2.3.3 $\tan(-23°)$

QUESTION 3

(GR 12 NSC November 2019)

3.1 In the diagram, the graphs of $f(x) = \sin x - 1$ and $g(x) = \cos 2x$ are drawn for the interval $x \in [-90° ; 360°]$. Graphs f and g intersect at A. B(360° ; −1) is a point on f.



P and Q are points on graphs g and f respectively such that PQ is parallel to the y −axis. If PQ lies between A and B, determine the value(s) of x for which PQ will be a maximum.

(GR 12 NSC March 2018)

3.2 Consider: $g(x) = -4\cos(x + 30°)$

3.2.1 Write down the maximum value of $g(x)$.

3.2.2 Determine the range of $g(x) + 1$.

3.2.3 The graph of g is shifted 60° to the left and then reflected about the x −axis to form a new graph h. Determine the equation of h in its simplest form.

(GR12 NSC NOV 2018)

3.3 Consider: $f(x) = -2\tan\frac{3}{2}x$

3.3.1 Write down the period of f.

3.3.2 The point A $(t ; 2)$ lies on the graph. Determine the general solution of t.

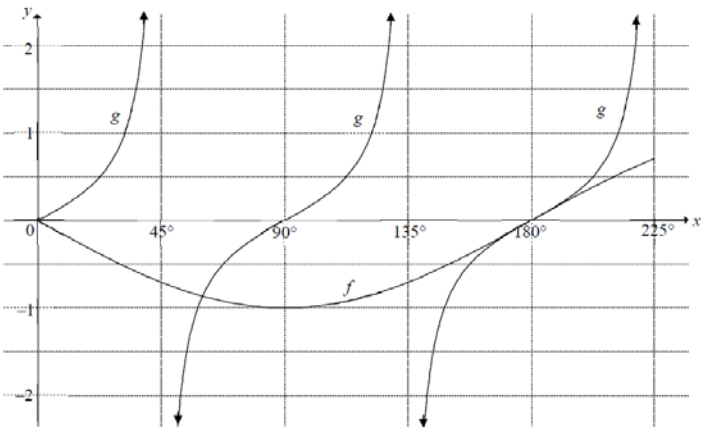
3.3.3 Draw the graph of f for the interval $x \in [-120 ; 180°]$.
Clearly show All asymptotes, intercepts with the axes and endpoint(s) of the graph.

3.3.4 Use the graph to determine for which values of x will $f(x) \geq 2$ for $x \in [-120 ; 180°]$.

3.3.5 Describe the transformation of graph f to form the graph of $g(x) = -2\tan(\frac{3}{2}x + 60°)$

(GR12 NSC March 2017)

3.4 In the diagram, the graphs of the functions $f(x) = a\sin x$ and $g(x) = \tan bx$ are drawn on the same system of axes for the interval $0° \leq x \leq 225°$.



3.4.1 Write down the values of a and b.

3.4.2 Write down the period of $f(3x)$.

3.4.3 Determine the values of x in the interval $90° \leq x \leq 225°$ for which $f(x).g(x) \leq 0$

QUESTION 4

(GR12 DBE 2018)

4.1 Determine the general solution of the following equation:

$$\sin(2x + 40°)\cos(x + 30°) - \cos(2x + 40°)\sin(x + 30°) = \cos(2x - 20°)$$

(GR12 DBE 2019)

4.2 Determine the general solution of $\cos(x - 30°) = 2\sin x$

(GR12 NSC NOV 2019)

4.3 Consider $f(x) = \sin(x + 25°)\cos 15° - \cos(x + 25°)\sin 15°$

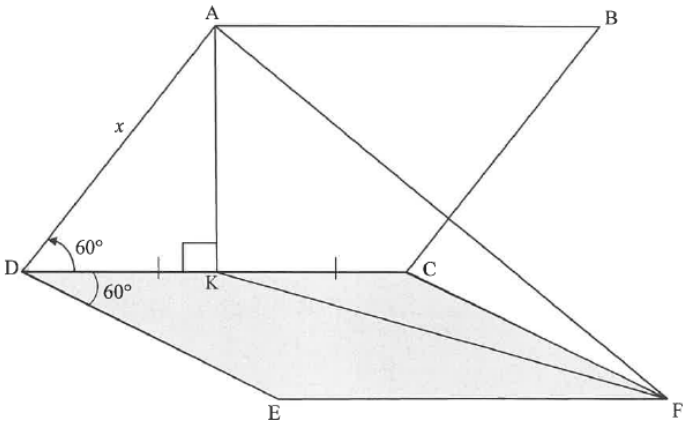
4.3.1 Determine the general solution of $f(x) = \tan 165°$

4.3.2 Determine the values(s) of x in the interval $x \in [0°; 360°]$ for which $f(x)$ will have a minimum value.

QUESTION 5

(GR12 NSC NOV 2019)

5.1 The diagram below shows solar panel, ABCD, which is fixed to a flat piece of concrete slab EFCD. ABCD and EFCD are two identical rhombuses. K, is a point on DC such that DK = KC and AK ⊥ DC. AF and KF are drawn. $A\hat{D}C = C\hat{D}E = 60°$ and AD = x units.



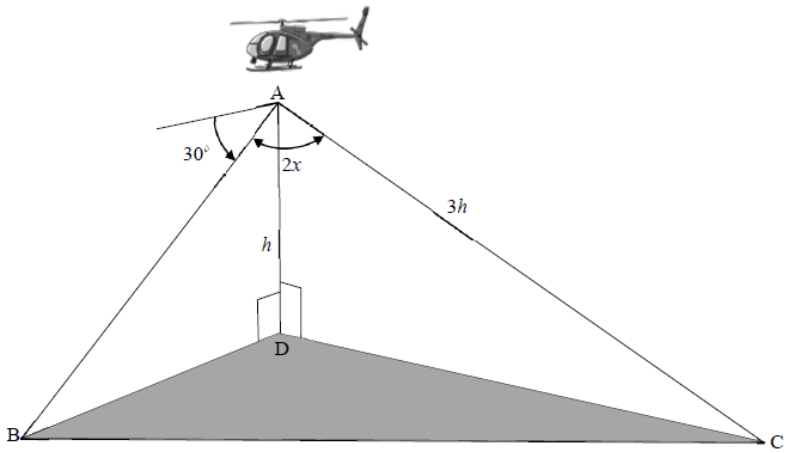
5.1.1 Determine AK in terms of x.

5.1.2 Write down the size of $K\hat{C}F$.

5.1.3 It is further given that $A\hat{K}F$, the angle between the solar panel and the concrete slab, is y. Determine the area of ΔAKF in terms of x and y.

(GR12 NSC NOV 2018)

5.2 A pilot is flying in a helicopter. At point A, which is h meters directly above point D on the ground, he notices a strange object at point B. The pilot determines that the angle of depression from A to B is 30°. He also determines that the control room at point C is $3h$ meters from A and $B\hat{A}C = 2x$. Points B, C and D are in the same horizontal plane.
This scenario is shown in the diagram below.



5.2.1 Determine the distance AB in terms of h.

5.2.2 Show that the distance between the strange object at point B and the control room at point C is given by BC= $h\sqrt{25 - 24\cos^2 x}$